STRESSES AND STRAINS ON THE SURFACE OF ANCHORS

Contraintes et déformations à la surface d'ancrage

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SOMMAIRE

Des essais d'arrachement d'ancrages de sol de petit diamètre (5 à 15 cm, longueur 3 m) ont donné des résistances limites élevées que l'on ne peut pas expliquer à partir de la pression du poids des terres.

On a exécuté des essais à grande échelle, dans lesquels la résistance au frottement ($\tau_{rz} = - \tau_{zr}$) et la contrainte normale (σ_r) à la surface (rugeuse) de l'ancrage ont été mesurées à l'aide de cellules. Les rapports entre les efforts de cisaillement et les contraintes normales ont donné des angles de frottement très élevés, qu'il n'est pas possible de déduire d'essais triaxiaux ou d'essais de cisaillement. De plus, des contradictions apparaissent si l'on tient compte des conditions cinématiques tout en retenant les hypothèses courantes au sujet de la coaxialité des contraintes et des déformations. Par contre, si on abandonne l'hypothèse de coaxialité tout en prenant en compte la dilatance des matériaux dans les conditions de cinématique existantes, l'analyse des cercles de Mohr pour les contraintes et les déformations permet d'éliminer toute contradiction dans les résultats des essais. Ces analyses théoriques prouvent que les effets de dilatation sont le facteur prépondérant qui détermine la force limite d'ancrages cylindriques.

On explique les modifications que l'on a été conduit à apporter aux essais classiques en laboratoire et les résultats ainsi obtenus confirment les considérations énoncées ci-dessus.

On discute finalement les conséquences fondamentale qui en résultent pour le calcul de la force limite d'éléments d'ancrage cylindriques.

SUMMARY

Pull-out resistance tests on cylindrical ground anchors with small diameters (5 to 15 cm) show a high bearing capacity not derivable from the overburden pressure.

To investigate this fact full scale tests were performed measuring skin friction ($\tau_{rz} = - \tau_{zr}$) and normal stress (σ_r) on the anchor surface (rough) with the help of installed cells. Analysing the ratio of skin friction and normal stress, large angles of internal friction were stated not to be derived from triaxial tests or from direct shear tests. Beyond that contradictions appear if kinematic conditions and usual assumption of coaxiality of stresses and strain increments are taken into account. However, if coaxiality is disregarded and using Mohr's stress and strain circles with regard to the kinematic conditions concerned, the test results do not point out any contradictions. These theoretical considerations reveal that dilatancy effects are above all responsible for the bearing capacity of cylindrical anchors.

Modifications of conventional laboratory tests are explained and test results of a «true direct shear tests» are given confirming the considerations stated above.

The fundamental impact of these results on the calculation of the bearing capacity of cylindrical anchors is discussed.

INTRODUCTION

Pull-out tests of cylindrical anchors with small diameters (5 to 15 cm) reveal a bearing capacity (as a consequence of skin friction) much higher than what would be expected from the overburden pressure (H. Zweck (1970), H.-U. Werner (1972), H. Ostermayer (1975)). Further in many experiments it is observed (T.H. Hanna (1973), B.M. Das (1975)) that the skin friction does not exceed a limiting value beneath a certain depth.

It is difficult to analyse these phenomena because there are a lot of factors influencing the bearing capacity of ground anchors such as:

- overburden pressure;
- boring technique (ramming or boring);
- diameter and inclination of anchors;
- bond length;
- pressure of cement injection;
- rate of penetration of cement suspension into soil;
- soil density;
- grain size and grain-size distribution of soil.

This variety led to simplified tests to separate the influence of soil parameters and to state the author's opinion that the pulling resistance of ground anchors is decisively influenced by dilatancy effects.

PULL-OUT TESTS

Large scale tests were performed to determine the pull-out resistance of vertical cylindrical anchors. They consisted of pieces of steel pipe connected inside with threaded steel bolts. The bolts were provided with strain gages to measure the skin friction for each of the pipe sections. The anchors (diameter $\approx$ 5 to 10 cm with rough surface) are held vertically in a round test bin (diameter 2.5 m, height 3 m, see E. Wernick (1974)) and the bin is then filled with sand, varying density from test to test.

At the anchor surface cells were inserted (fig. 1) to measure the skin friction ($\tau_{rz} = -\tau_{zr}$) and the normal stress (σ_r) at the same place. The details of the cells (with semiconductor strain gages) are given elsewhere (see B. Prange (1971)).

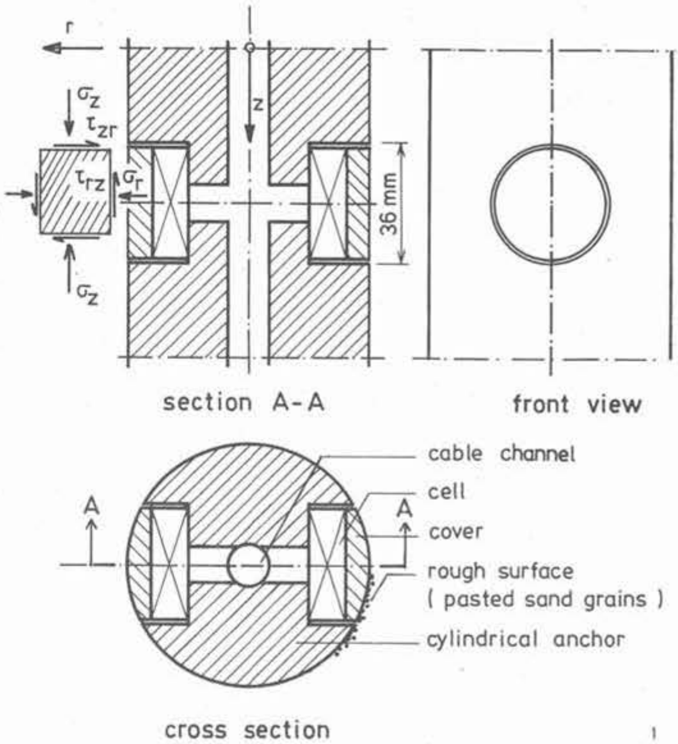

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Fig. 1. — **Arrangement of cells in the anchor.**

CELL MEASUREMENTS

A representative result of cell measurements, obtained at a depth of $z = 2.12$ m below ground surface, i.e., surface of the sand body, is shown in fig. 2. Point «1» represents the anchor erected in the centre of the test bin; all stresses here are zero (reference stadium). During sand filling $|\sigma_r|$ increases to about 10 kN/m² (0.10 kp/cm², «2») which is in good agreement with the earth pressure at rest and can be calculated as $\sigma_r = K_o \cdot \gamma \cdot z = (1 - \sin \Phi') \gamma \cdot z = -(1 - \sin 40°) \cdot 17.14\ \text{kN/m}^3 \cdot 2.12\ \text{m} = -13.0\ \text{kN/m}^2$. But one may not be deceived by this agreement, because σ_r is also influenced by «negative skin friction» caused by relative displacement between sand and anchor surface during filling, in this case $\tau_{rz} \approx -5$ kN/m² (-0.05 kp/cm²) was measured.

During the following pull-out test —after overcoming the «negative skin friction»— $|\sigma_r|$ and τ_{rz} increase

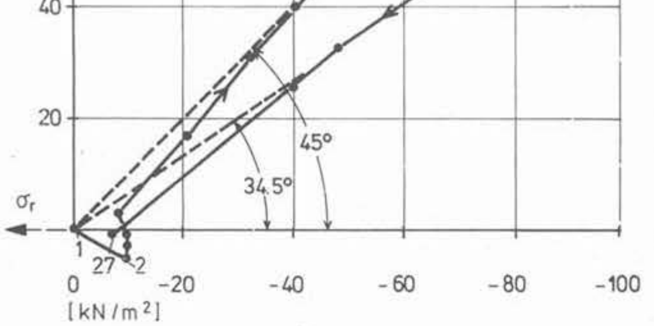

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Fig. 2. — **Typical stress path measured on the anchor surface during pull-out test.**

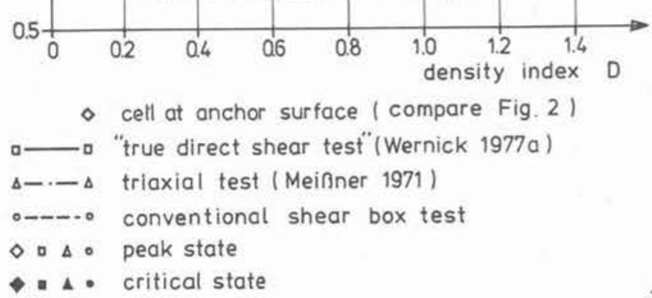

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Fig. 3. — **Angles of shearing resistance obtained in different types of shear tests on the same sand.**

rapidly until the maximum bearing capacity, Z_{max}, of the anchor is reached. During this period the angle of shearing resistance rises up to $\Phi'_p = 45°$ (p = peak). Continuing the test strain-controlled («17» to «23») Φ' dropped to a critical value $\Phi'_k = 34.5°$. After relieving («27») skin friction vanished and $\sigma_r \approx -8$ kN/m^2 ($-$ 0.08 kp/cm^2) corresponded to the earth pressure at rest again.

Two main points should now be noted:

- the increasing $|\sigma_r|$ clearly shows the wedging effect arising **during** pull-out test because only the pressure at rest existed before starting ;
- very large angles of shearing resistance appeared.

The latter is not confirmed if these angles are compared with the results of the conventional shear box and triaxial tests conducted on the same sand (fig. 3). In fig. 2 the initial density index was D = 1.1

(D = $(\gamma - \gamma_l)/(\gamma_d - \gamma_l)$, γ_l unit weight in «loosest», γ_d in «densest» state packing, see DIN 18 126).

THE «TRUE DIRECT SHEAR APPARATUS»

This discrepancy and the appearance of a shear band close to the anchor surface (pointed out in E. Wernick (1977 a)) led to the development of a «true direct shear apparatus» (fig. 4). The primary modification to the conventional box shear apparatus is that the loading platen is not permitted to turn, but can move in the vertical direction only. Three typical results of the «true direct shear test» are given in fig. 5.

During horizontal displacement h —enforcing a horizontal shear band inside the sample— the angle of shearing resistance reaches a peak value Φ'_p and a residual one Φ'_k (fig. 5 a). Φ'_k corresponds to a critical

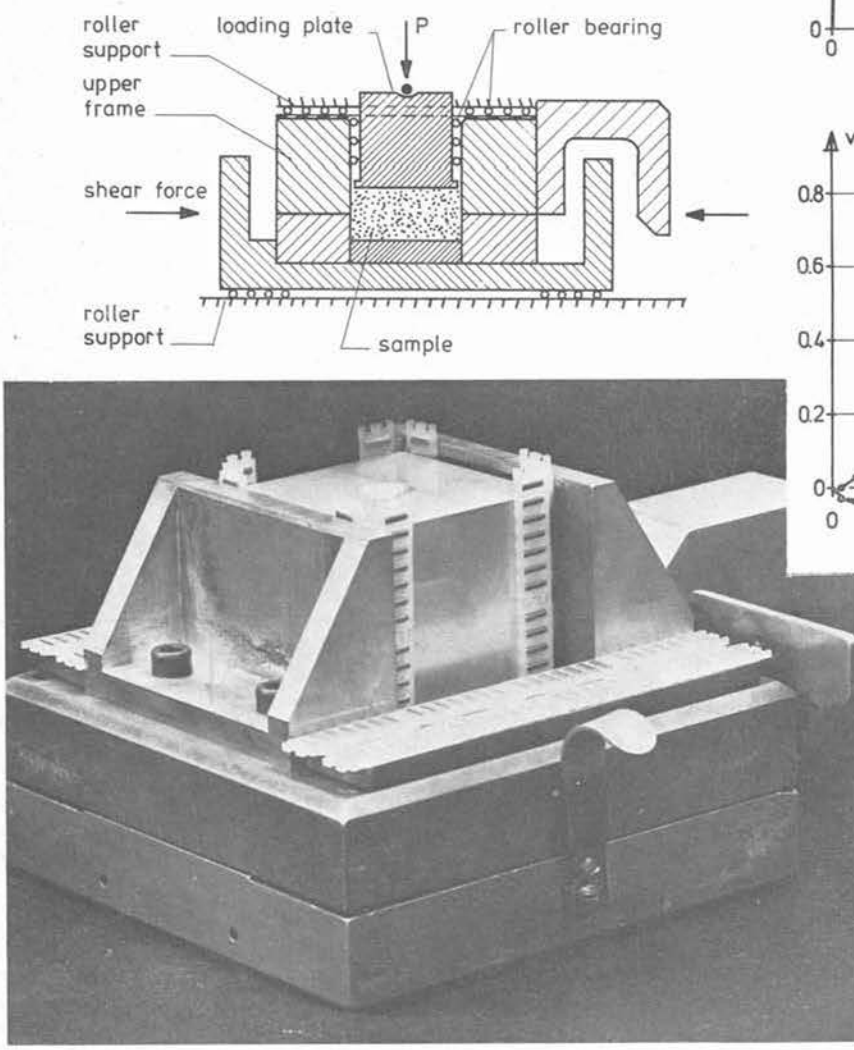

Fig. 4. — **Cross section and picture of the « true direct shear apparatus ».**

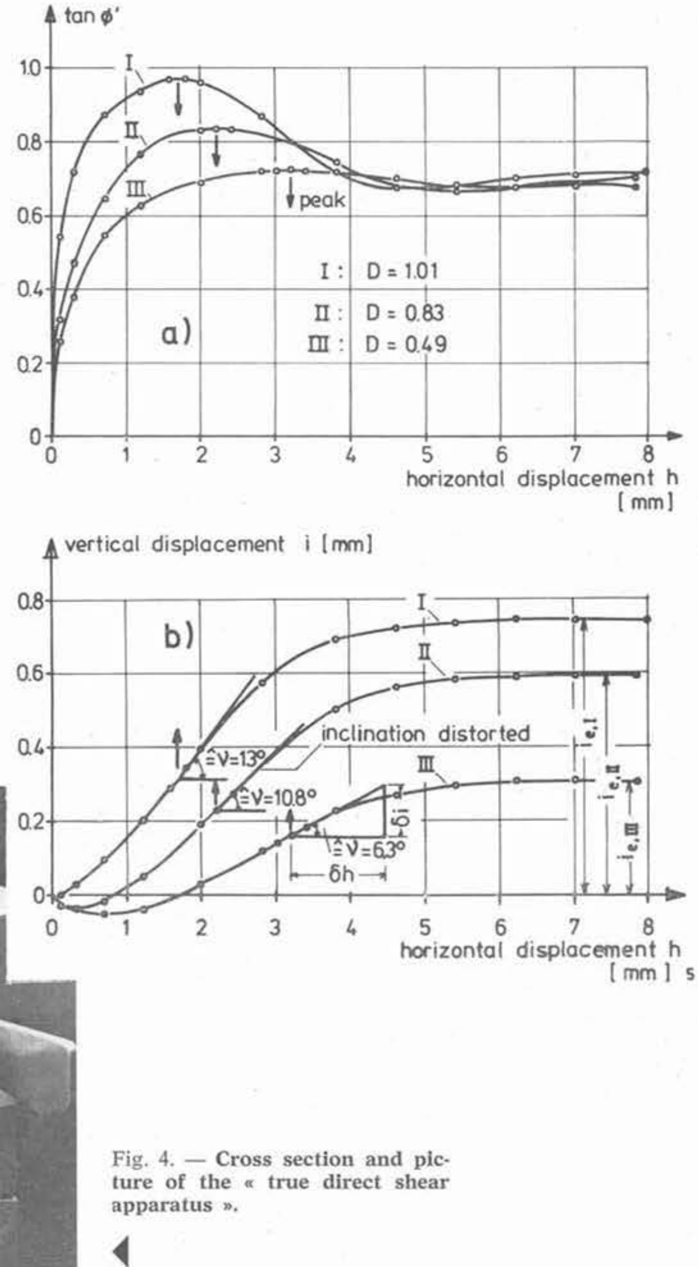

Fig. 5. — **Results of « true direct shear tests ».**

density index inside the shear band not being influenced by the initial density index. Transfering these Φ'_p and Φ'_k values into fig. 3 and comparing especially Φ'_p with results of triaxial and conventional direct shear tests, a great discrepancy is observed again. But there is a good agreement between Φ'_p and Φ'_k measured by the cell (see rhombus in fig. 3).

The different values of Φ'_p and Φ'_k are to be explained by the varying kinematical conditions of the different shearing procedures: on the anchor surface and inside the shear band the kinematical conditions are more restricted than in triaxial tests (strains in two directions) and conventional shear box tests (turning of loading plate allows additional strains).

Only a few types of direct shear apparatusses avoid this additionally kinematical freedom as the direct shear machine with an automatic recorder, C.R.M. Maguin (1956) and likewise the improved direct shear apparatus of J.R.F. Arthur et al. (1977).

STRESS AND STRAIN CONDITIONS

It is not desirable to consider the stress and strain conditions on the anchor surface at first because strain increments are unknown except $\delta\,\varepsilon_z$ being zero (δ = increment) assuming that the anchor is rigid and that the failure occurs along a shear band appearing at the anchor surface.

«TRUE DIRECT SHEAR TEST»

In contrast to this the strain conditions of the «true direct shear test» are known (and are comparable with those of the anchor tests as to be seen later):

- the failure occurs along a shear band following a zero extension line i.e. $\delta\,\varepsilon_x = 0$ (fig. 6 a);
- assuming that all deformations are confined to the shear band the angle of dilatancy ν can be calculated from fig. 5 b by $\tan\nu = \delta i/\delta h$ and $\delta\,\varepsilon_y = \delta i/s$, s = thickness of shear band;
- the third strain $\delta\,\varepsilon_2$ (marked with 2 to distinguish it from z in fig. 1) is zero (plane strain).

Using this information Mohr strain circles can be drawn. Test I from fig. 5 was chosen to draw the Mohr circles in fig. 6 b and 6 e. Fig. 6 b has been drawn for the stadium of peak friction, ν being 13° at this stage (fig. 5 b). Fig. 6 e represents the strain circle for residual stadium, ν being zero here.

To draw the Mohr stress circles a further assumption has to be made, because only σ_y (vertical stress) and τ_{yx} (shear resistance) are known. The usual assumption in soil mechanics is coaxiality of stresses and strain increments —yielding dotted circles in fig. 6 d (peak state) and fig. 6 (residual state). According to the Mohr-Coulomb criterion the plane of maximum stress obliquity gives an angle of peak shearing resistance $\Phi''_p = 53.1°$ and a residual one of $\Phi''_k = 41.7°$. Comparing $\tan\,\Phi_p = 1.33$ (D = 1.1) and $\tan\,\Phi''_k = 0.89$ with the results of shear tests in fig. 3, it is obvious that the assumption of coaxiality cannot hold in this case.

Instead of this we now make the assumption, that the failure follows a static characteristic i.e. a plane of maximum stress obliquity. In this way one can get the solid circles in fig. 6 d and 6 f. The angles of shearing resistance measured directly in the «true direct shear test» now correspond to the inclination of the tangents to these Mohr circles and the above contradictions vanish. The deviation of the principal axes of strain increments and stresses is $(45° - \nu/2) - (45° - \Phi'/2) = \Phi'/2 - \nu/2$ (the deviation would be zero, if coaxiality were valid).

It would be desirable to compare these somewhat unusual results with results of other investigations particularly with respect to

- large angles of shearing resistance;
- the assumption of non-coaxiality.

Peak angles of shearing resistance of the same magnitude were found by J. Vardoulakis (1977). who performed plane strain tests with the same sand. This agreement is at first surprising since the stress strain conditions in these tests run coaxial up to the peak differing from those in «true direct shear». Hence the conclusion may be drawn that the Mohr-Coulomb criterion is valid in both cases with the same angle of shearing resistance irrespective of the deviation of the principal axes of strain increments from those of the stresses.

The investigations of J. Vardoulakis (1977) also confirm the assumption of non-coaxiality in shear bands under certain kinematic conditions. They prove theoretically (conceiving the formation of shear bands as a bifurcation problem) and experimentally that the non-coaxiality holds inside a shear band under kinematic conditions as is the case here.

J.R.F. Arthur et al. (1977) also found developing an empirical model and discussing data from different shear apparatuses that principal axes of stress and strain increment do not always coincide. They prove that the deviation of principal axes of stress and element forming thin rupture layers. For kinematical conditions as discussed here they also found a deviation as given in the last chapter.

ANCHOR

We can now consider the stress and strain conditions near the anchor surface again. Starting from the appearence of a shear band near the anchor surface we have to estimate the deviation of the actual strain state from plane strain. In doing this we assume that the first invariant of strain increments (volume change) in «true direct shear» (t.d.s.)

$$I_{\delta\varepsilon,\,tds} = \delta\,\varepsilon_x + \delta\,\varepsilon_y + \delta\,\varepsilon_2 = \delta i/s, \quad \text{(fig. 6 a)} \tag{1}$$

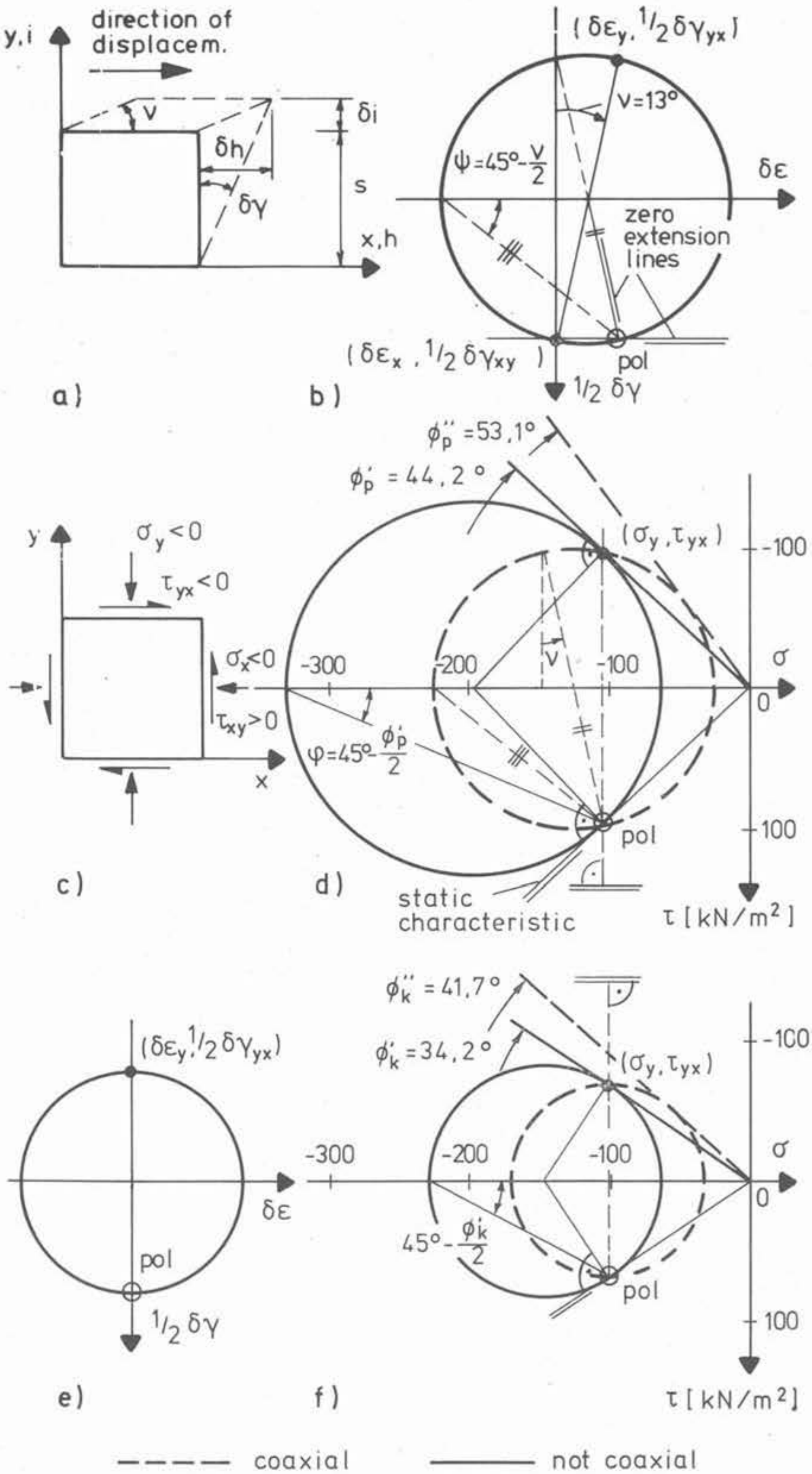

Fig. 6. — **Stress and strain analysis in the shear band.**

is equal to the volume change inside the shear band near the anchor surface (a)

$$I_{\delta\varepsilon,a} = \delta\varepsilon_r + \delta\varepsilon_t + \delta\varepsilon_z = \frac{d\,\delta u}{dr} + \frac{\delta u}{r}, \text{ (fig. 7)} \quad (2)$$

Assuming that the first invariant of stresses I_σ has no influence on $I_{\delta\varepsilon}$ or is of the same magnitude (sign convention of continuum mechanics used: for traction and extension positive signs) and setting

$$I_{\delta\varepsilon,\,tds} = I_{\delta\varepsilon,\,a} \quad (3)$$

we obtain the differential équation

$$\frac{d\,\delta u}{dr} + \frac{\delta u}{r} = \frac{\delta i}{s} \quad (4)$$

whose solution with the boundary condition

$$\delta u\,(r_a) = 0 \tag{5}$$

(r_a = radius of anchor, fig. 7) is

$$\delta u = \frac{\delta i \left(r - \frac{r_a^2}{r}\right)}{2\,s}. \tag{6}$$

Taking now the ratio $\delta\,\varepsilon_t/\mathrm{Dev}_{\delta\varepsilon,\mathrm{tds}}$, $\mathrm{Dev}_{\delta\varepsilon,\mathrm{tds}}$ being the deviator of strain increments of the «true direct shear test», as a measure for the deviation from plane strain we obtain.

$$\frac{\delta\,\varepsilon_t}{\mathrm{Dev}_{\delta\varepsilon,\mathrm{tds}}} = \frac{\frac{\delta i}{2\,s}\left(1 - \frac{r_a^2}{r^2}\right)}{\frac{\sqrt{2\,\delta i^2 + \frac{3}{2}\,\delta h^2}}{\sqrt{3}\,.\,s}} = \tag{7 a}$$

$$= \frac{\sqrt{3}\tan\nu\left(1 - \frac{r_a^2}{r^2}\right)}{2\,.\sqrt{2\tan^2\nu + 1.5}} \tag{7 b}$$

where

$$\frac{\delta i}{\delta h} = \tan\nu,\ \text{(fig. 6 a)}\,. \tag{8}$$

In order to make a quantitative estimate of the ratio in (7) it is necessary to set a limiting value r_a/r and thus for r_a/s. For reasons explained elsewhere (see E. Wernick (1977 a)) the ratio of anchor diameter to thickness of the shear band is limited to

$$2\,r_a/s \geq 10\,. \tag{9}$$

Considering middle of the shear band $r = r_a + s/2$ and inserting $\tan\nu = 0.25$ as the most unfavourable value, we obtain.

$$\delta\,\varepsilon_t/\mathrm{Dev}_{\delta\varepsilon,\mathrm{tds}} = 3\%\,. \tag{10}$$

Even for these unfavourable conditions the deviation of the strain state inside the shear band from plane strain is very small and therefore negligible. Further it is interesting to consider three limiting cases in equation (7 b):

- dilatation or $\tan\nu$ is zero (for residual case for example);
- r approaches r_a and
- r_a is very large (with $r = r_a + s$ the ratio $r_a/r \to 1$).

For all these cases the plane strain condition is achieved.

These considerations reveal that results of the «true direct shear test» can be used to calculate bearing capacity of cylindrical anchors. Neglecting the above considered deviation of plane strain they permit determination of

- the correct angle of shearing resistance;
- the correct angle of dilatancy and
- the magnitude of dilatation inside the shear band from (6), which for $r = r_1$ (fig. 7) is

$$\delta u_1 = \delta i\ \frac{r_a + \frac{s}{2}}{r_a + s} \tag{11}$$

or at the end of pull-out test (in total) is

$$u_1 = i_e\ \frac{r_a + \frac{s}{2}}{r_a + s}. \tag{12}$$

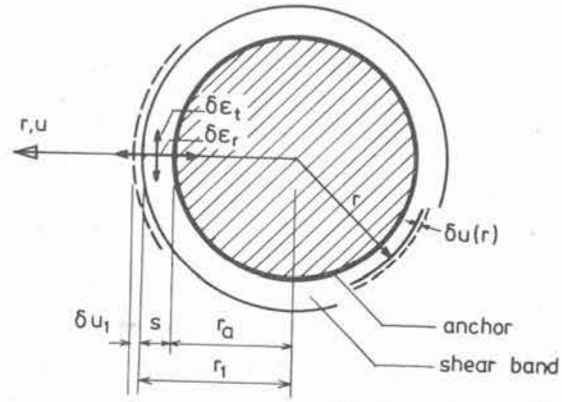

Fig. 7. — **Cross section of anchor with shear band, notations.**

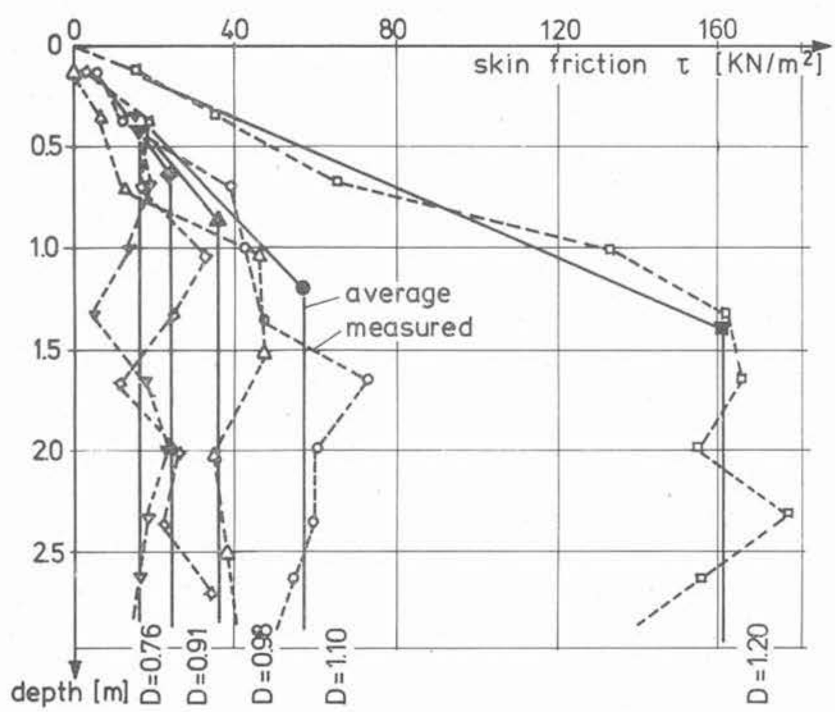

Fig. 8. — **Distribution of skin friction at maximum bearing capacity (anchor diameter 7.42 cm).**

For i_e in relation to initial density see fig. 5. The thickness of shear band for the sand used here is $s \approx 15 . d_{50}$, d_{50} being the average grain size, E. Wernick (1977 b).

Equation (12) quantifies the wedging effect in a very simple manner and it is now obvious that dilatancy causes skin friction not being a function of the overburden pressure. This is stated also by results of pull-out tests given in fig. 8.

We remark that the depth of initial constant skin friction is a function of its magnitude or of soil density index, which is not discussed in more detail here. Further more, from the distances of the average lines in fig. 8 it can be concluded —with respect to written density index D— that skin friction increases more than linear with increasing density index. But considering the influence of density index it has to be remembered that the test having D = 1.20 was compacted by vibration whereas the others were built up by sand rain.

All these considerations reveal that a theoretical model to calculate the skin friction has to take into account the appearance of a shear band near the anchor surface and wedging effect caused by dilatancy. Such a model using results of « true simple shear tests » is under preparation and will be published elsewhere.

CONCLUSION

Analysing the stress and strain conditions on the surface of cylindrical anchors it was possible to prove that the high bearing capacity is caused by a wedging effect produced by dilatation. Furthermore large angles of shearing resistance were measured with the help of cells installed on the surface of anchors, which could not be confirmed by conventional shear tests. These values could, however, be confirmed by the test results of a «true direct shear apparatus». The apparatus also permits the determination of the angle of dilatancy inside a shear band.

Analysing stress and strain conditions of the «true direct shear test» it was pointed out that the usual assumption of coaxiality does not hold. Furthermore, the results of «true direct shear test» are transfered to the anchor with help of invariants proving that the deviation of plane strain is negligible in the shear band. Wedging effect could thus be quantified and it is now obvious that skin friction is not a linear function of the overburden pressure but is mainly a consequence of wedging effect, being a geometrical effect. This led to the judgement that a reasonable ratio of grain size or thickness of shearband to diameter of anchor or corresponding measure has to be observed for model tests in granular materials producing stresses as consequence of hindered deformations. Above that the compressibility of the surrounding soil has to be imitated correctly. To avoid such difficulties the problem under discussion here was investigated in full scale.

The «true direct shear test» is of much more importance far beyond the anchor problem. Correct soil parameters can be ascertained for all problems where shear bands of corresponding kinematics appear or can be assumed. The higher shear resistance actually available could be used to obtain economic design in soil and foundation engineering. For this reason the introduction of the «true direct shear test» as a routine test is recommended. It is shown that this can be carried out by a simple reconstruction of conventional shear box apparatuses.

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